For the example with Latin square design of 4 row and 4 columns and 4 treatments (A - D), the allocation of treatment to rows and columns can be presented in the following table

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Row | Column | | | |
| 1 | 2 | 3 | 4 |
| 1 | A | B | C | D |
| 2 | D | A | B | C |
| 3 | C | D | A | B |
| 4 | B | C | D | A |

Giving that s2, sR2 and sC2 denote the measurement error, between rows and between columns variance component, respectively, and theta is the variance component corresponding to the fixed treatment effects. The theoretical ANOVA table generated from infoDecompuTE can be written as

|  |  |  |
| --- | --- | --- |
| Source of variation | DF | EMS |
| Bw Rows | 3 | s2 + 4\*sR2 |
| Bw Cols | 3 | s2 + 4\*sC2 |
| Bw Rows.Cols | 9 |  |
| Treatment | 3 | s2 + theta |
| Residual | 6 | s2 |

For a different example with Youden rectangle design of 4 row and 3 columns and 4 treatments (A - D), the allocation of treatment to rows and columns can be presented in the following table

|  |  |  |  |
| --- | --- | --- | --- |
| Row | Column | | |
| 1 | 2 | 3 |
| 1 | A | B | C |
| 2 | D | A | B |
| 3 | C | D | A |
| 4 | B | C | D |

The theoretical ANOVA table generated from infoDecompuTE can be written as

|  |  |  |  |
| --- | --- | --- | --- |
| Source of variation | DF | EMS | *E* |
| Bw Rows | 3 |  |  |
| Treatment | 3 | s2 + 3\*sR2 | 1/9 |
| Bw Cols | 2 | s2 + 4\*sC2 |  |
| Bw Rows.Cols | 6 |  |  |
| Treatment | 3 | s2 + theta | 8/9 |
| Residual | 3 | s2 |  |

 For a 2-by-3 factorial experiment arranged in a 6-by-6 row-column design, where the upper case letter denotes the treatment 1 and lower case letter denotes treatment 2, the allocation of these two treatments to row and column can be written as

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Row | Column | | | | | |
| 1 | 2 | 3 | 4 | 5 | 6 |
| 1 | Aa | Bb | Ac | Ba | Ab | Bc |
| 2 | Bb | Ac | Ba | Ab | Bc | Aa |
| 3 | Ac | Ba | Ab | Bc | Aa | Bb |
| 4 | Ba | Ab | Bc | Aa | Bb | Ac |
| 5 | Ab | Bc | Aa | Bb | Ac | Ba |
| 6 | Bc | Aa | Bb | Ac | Ba | Ab |

Giving that theta1, theta2 and theta12 denote the variance component corresponding to the fixed effects of treatment 1, treatment 2 and interaction. The theoretical ANOVA table generated from infoDecompuTE can be written as

|  |  |  |
| --- | --- | --- |
| Source of variation | DF | EMS |
| Bw Rows | 5 | s2 + 6\*sR2 |
| Bw Cols | 5 | s2 + 6\*sC2 |
| Bw Rows.Cols | 25 |  |
| Treatment1 | 1 | s2 + theta1 |
| Treatment2 | 2 | s2 + theta2 |
| Interaction | 2 | s2 + theta12 |
| Residual | 20 | s2 |

The following example describes the case where treatment 1 is confounded with rows. The confounding is arise where Row 1 to 3 contain only Treatment A and Row 4 to 6 contain only Treatment B,

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Row | Column | | | | | |
| 1 | 2 | 3 | 4 | 5 | 6 |
| 1 | Aa | Ab | Ac | Aa | Ab | Ac |
| 2 | Ab | Ac | Aa | Ab | Ac | Aa |
| 3 | Ac | Aa | Ab | Ac | Aa | Ab |
| 4 | Ba | Bb | Bc | Ba | Bb | Bc |
| 5 | Bb | Bc | Ba | Bb | Ac | Ba |
| 6 | Bc | Ba | Bb | Bc | Ba | Bb |

The theoretical ANOVA table generated from infoDecompuTE can be written as

|  |  |  |
| --- | --- | --- |
| Source of variation | DF | EMS |
| Bw Rows | 5 |  |
| Treatment1 | 1 | s2 + 3\*sR2 + theta1 |
| Residual | 4 |  |
| Bw Cols | 2 | s2 + 4\*sC2 |
| Bw rows.Cols | 25 |  |
| Treatment2 | 2 | s2 + theta1 |
| Interaction | 2 | s2 + theta12 |
| Residual | 21 | s2 |

**Crossing between the Phase 1 and Phase 2 block structures**

My current understanding in the relationship between the Phase 1 and Phase 2 block structures is deduced from the decomposition procedure by treating the Phase 1 block structure as the treatment structure. The method follows by checking for any block information of the Phase 1 block structure in each stratum defined from the Phase 2 block structure. Hence, the Phase 2 block structure always contribute to the outer stratum of the ANOVA table, whereas the Phase 1 block structure always contribute to the inner stratum of the ANOVA table.

For the example of the metabolomics experiment, if the Phase 2 block structure is Day\*Period and Phase 1 block structure is Animal, the method first defined the outer strata from the Phase 2 block structure, which are Between Days, Between Periods, Between Days and Periods and Within Days and Periods. The decomposition of the Phase 1 block structure in the Phase 2 block structure is check to define the Between Animals and Within Animals strata is each of four strata of the Phase 2 block structure. The decomposition of Phase 1 block structure in the Phase 2 block structure can be presented in the following table,

|  |  |
| --- | --- |
| Phase 2 block structure | Phase 1 block structure |
| Between Days | Between Animals |
|  | Within Animals |
| Between Periods | Between Animals |
|  | Within Animals |
| Between Days and Periods | Between Animals |
|  | Within Animals |
| Within Days and Periods | Between Animals |
|  | Within Animals |

In addition, since each day, period and day-period combination contains all six animals, the animal is orthogonal to all the block structure of Phase 2 experiment and Between Animals and Within Animals stratum can only be seem in the Within Days and Periods stratum. Hence, the previous table can be simplified to

|  |
| --- |
| Overall block structure |
| Between Days | |
| Between Periods | |
| Between Days and Periods | |
| Within Days and Periods | |
| Between Animals | |
| Within Animals | |

If there are crossing between the every block factor from the Phase 1 and Phase 2 experiments, then the overall block structure becomes Between Days, Between Periods, Between Days and Periods, Between Animals, Between Animals and Days, Between Animals and Periods, Between Animals and Days and Periods. Compare this block structure of the previous, only the Between Animals and Days and Between Animals and Periods are now presented, which is resulting from the crossing relationship between the Animal factor to Day and Period factors. These are then hidden in the Within Animals Within Days and Periods stratum.

One way to make these two strata appear in the ANOVA table is to fit these two interaction effects, Animal:Day and Animal:Period, in the Phase 1 block structure, i.e.

summaryAovTwoPhase(doePhase2.df, blk.str1 = "Animal + **Animal:Day + Animal:Period**", blk.str2 = "Day\*Period", trt.str = "Protocol\*Lobe")

**Result from aov**

Error: Day

Df Sum Sq Mean Sq F value Pr(>F)

Residuals 2 29.53 14.76

Error: Period

Df Sum Sq Mean Sq

Protocol:Lobe 1 17.63 17.63

Error: Animal

Df Sum Sq Mean Sq F value Pr(>F)

Protocol 1 8.51 8.507 0.5 0.518

Residuals 4 68.04 17.009

Error: Day:Period

Df Sum Sq Mean Sq F value Pr(>F)

Protocol:Lobe 1 0.0656 0.0656 0.043 0.87

Residuals 1 1.5302 1.5302

Error: Day:Animal

Df Sum Sq Mean Sq F value Pr(>F)

Lobe 2 0.77 0.383 0.022 0.978

Protocol:Lobe 2 5.95 2.973 0.173 0.845

Residuals 6 103.14 17.191

Error: Period:Animal

Df Sum Sq Mean Sq

Lobe 3 110.70 36.90

Protocol:Lobe 2 46.51 23.26

Error: Day:Period:Animal

Df Sum Sq Mean Sq F value Pr(>F)

Lobe 3 7.05 2.349 0.121 0.944

Protocol:Lobe 2 48.10 24.050 1.243 0.365

Residuals 5 96.73 19.347

**Result from summaryAovTwoPhase**

$ANOVA

DF e Animal:Period Animal:Day Animal Day:Period Period Day

Between Day

Between Animal:Day 2 1 0 2 0 6 0 12

Between Period

Between Animal:Period

Protocol:Lobe 1 1 3 0 0 6 18 0

Between Day:Period

Residual

Protocol:Lobe 1 1 0 0 0 6 0 0

Residual 1 1 0 0 0 6 0 0

Within

Between Animal

Protocol 1 1 3 2 6 0 0 0

Residual 4 1 3 2 6 0 0 0

Between Animal:Day

Lobe 2 1 0 2 0 0 0 0

Protocol:Lobe 2 1 0 2 0 0 0 0

Residual 6 1 0 2 0 0 0 0

Between Animal:Period

Lobe 3 1 3 0 0 0 0 0

Protocol:Lobe 2 1 3 0 0 0 0 0

Residual

Lobe 3 1 0 0 0 0 0 0

Protocol:Lobe 2 1 0 0 0 0 0 0

Residual 5 1 0 0 0 0 0 0

$EF

eff.Protocol eff.Lobe eff.Protocol:Lobe

Between Day

Between Animal:Day

Between Period

Between Animal:Period

Protocol:Lobe 1/9

Between Day:Period

Protocol:Lobe 8/9

Within

Between Animal

Protocol 1

Between Animal:Day

Lobe 1

Protocol:Lobe 1

Between Animal:Period

Lobe 2/9

Protocol:Lobe 4/9

Residual

Lobe 40/63

Protocol:Lobe 5/9